Calendars, Math, and Data Science: A Non-Western and Historical Exploration

Credit: NASA DSCOVR

https://sweetmath.rocks

TEACHER'S NOTES

Goals:

- Problem solving
- Math used in the world
- Interdisciplinary connection between Astronomy and Math
- Using a spreadsheet program, using formulas within it

Initial Question:

How have different calendars dealt with the fact that Earth travels around the sun in 365.2422 days? (This decimal has been quite problematic.)

Before You Begin:

If you would like your students to research the calendars themselves, we recommend that students work in groups, use the Gregorian calendar, and choose one other calendar with their group. If you would like students to do this research, print page 8, not page 9. The readings about the calendars become quite complex. To avoid the difficult research, you may print out Ms. Sweet's Notes (page 9) for all students. Page 8, the blank copy of the chart, may be eliminated in that case.

If you would like all students to get the same answers, you will need to tell them which year of the leap year cycle to begin with (i.e. start with leap year for the Gregorian calendar). If you do not tell them this, they will get different answers, and it will provide a great way to dive into one another's work and use their investigative minds. You may know what your class needs: shared answers or a debate.

Supplies:

- One computer/tablet per group (for spreadsheet software program)
- One printed packet for each group (pages 5- 8, 9 or 10, 11), stapled together, double-sided ok
- Class set of page 12, separate
- Some copies of Extension, page 17

Remembering Astronomy:

You may wish to begin with a class review of how/why the moon appears to wax and wane as it orbits Earth, and how the sun determines the seasons as Earth orbits the sun. We recommend the following resources:

NASA's "What causes the seasons?" Available at https://spaceplace.nasa.gov/seasons/en/ Accessed 2/15/2020.

NASA's "The Moon's Phases in Oreos: Why does the moon look different throughout the month?" Available online at https://spaceplace.nasa.gov/oreo-moon/en/ Accessed 2/15/2020.

Some Favorite Resources for Teachers:

"No perfect calendar: why we have leap years and why the Earth's always out of sync" by Zulfikar Abbany. Available online https://www.dw.com/en/no-perfect-calendar-why-we-have-leap-years-and-why-the-earths-always-out-of-sync/a-19077064

"Calendar Calculations" by NASA. Available online https://www.grc.nasa.gov/www/k-12/Numbers/Math/Mathematical_Thinking/calendar_calculations.htm

See page 7 for a more extensive list.

TEACHER'S NOTES

The Lesson:

1. Hand out the student packet (pages 5-8, 9 or 10, 11). Students will read the pages aloud as a group or silently to themselves.

2. Tell students how many calendars you expect them to compare. If you would like all students to get the same answers, tell them to start with a leap year in the Gregorian calendar (and any other calendar without a fixed cycle), or tell them to all start with the first year of the cycle. You may wish to give students some time (5-20 minutes) to try page 11 without the chart.

3. Hand out page 12 with the chart on it to give students structure.

4. You may wish to tell them to get computers out and share a Google Sheets link with them, an Excel file with them, or have them try the paper first.

5. We recommend that students use a spreadsheet program on a computer to avoid addition errors and help keep accurate records of their work.

6. When students get to the part where they need to add 19 years for a calendar, ask if they know the formula for adding in the spreadsheet program. Often it is something like this: =SUM(B2:B20).

	A	G
1	Year #	Gregorian
2	1	365
3	2	365
4	3	365
5	4	366
6	5	365
7	6	365
8	7	365
9	8	366
10	9	365
11	10	365
12	11	365
13	12	366
14	13	365
15	14	365
16	15	365
17	16	366
18	17	365
19	18	365
20	19	365
21	Total days in 19 years	=SUM(G2:G20)

7. Some students will retype this formula into the next column. The teacher or a peer can show them how to click and drag the formula across and have it copy the formula for the whole row. Sometimes we do this on the projector for the entire class to see; other times we walk around the room, showing each group. When we show students this trick on their own computer, we then delete our help and ask them to show us how they can do it themselves. This helps students feel ownership over their project.

8. Once most students have completed their chart, they will need to decide which calendar is most accurate (closest to Earth's actual revolution around the sun). They can use a formula for this as well, using the $ symbol to pin the Earth's revolution total minus the calendar's total. For example:

SUM | =$H$21-C21

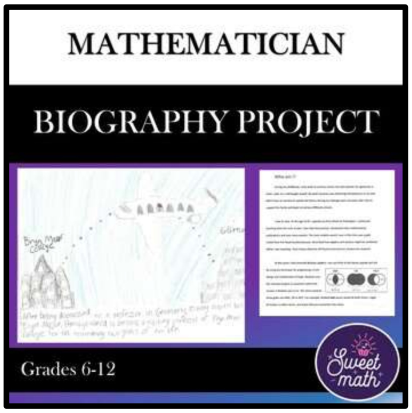
	A	B	C	D	E	F	G	H
1	Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
2	1	365	354	354	354	354	365	365.2422
3	2	365	354	354	354	354	365	365.2422
4	3	365	384	384	383.5	354	365	365.2422
5	4	365	354	354	354	354	366	365.2422
21	Total days in 19 years	6939.75	6936	6936	6932.5	6726	6939	6939.6018
22	Difference from Earth's revolution	-0.1482	=$H$21-C21					

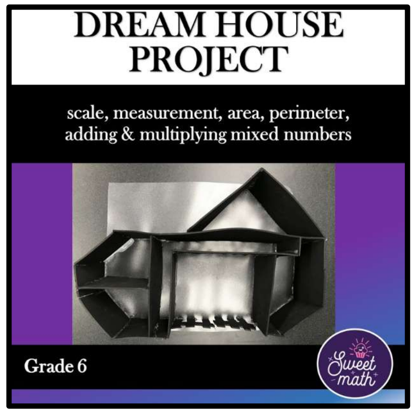
This formula can then be clicked and dragged across the row to copy. If students do not use $ to pin the Earth's revolution, the rest of their answers will become the same as the calendars' totals: e.g. 6939. This will probably flag as an error for them, but it may not. They will probably need a mini lesson on using the $. We recommend that you deliver this lesson after they have experienced the problem, so they understand its usefulness.

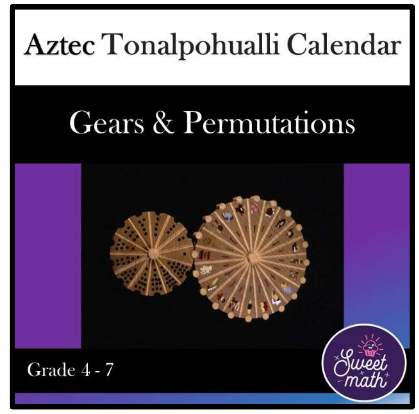
9. Once several groups have their answers, ask them to compare with a neighboring group. If they did not all begin their leap year cycles the same way, they will have different answers than some other groups, and a debate will ensue. We recommend allowing students to discuss among themselves for a time, then bringing the debaters to a projector, showing student work to the whole class, and leading a somewhat formal debate in front of the class.

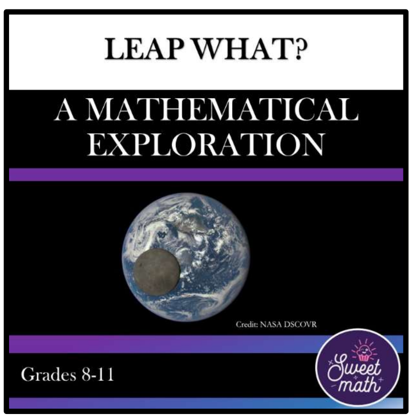
10. If a group or two become satisfied with their answers after meeting with another group, and you are also satisfied with their explanations and written work, you may hand them Extension page 16.

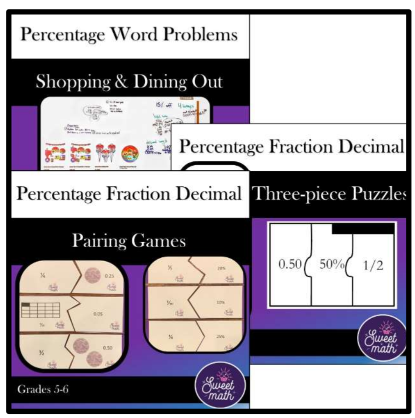
YOU MAY ALSO LIKE...











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CALENDARS & MATH

Name: ________________

Date: ________________

How have different calendars dealt with the fact that Earth travels around the sun in 365.2422 days? (This decimal has been quite problematic.)

First, we'll think about how we humans came to think about time and calendars. When the sun goes up, we call it a new day. When the sun goes down, moonlight (sometimes) appears. This change in light has become central to all human beings' understanding of time. Some cultures call sundown the beginning of a day; others use a different system (like midnight is the hour zero).

Some groups of people have used the moon to keep track of time as well, creating months based on the waxing and waning of the moon (29.5 days due to the moon's orbit around Earth). These are called lunar months (*luna* meaning "moon"). Some cultures track from full moon to full moon, while others track from new moon to new moon. Some cultures use two 15-day periods: one for waxing, one for waning.

Every society can measure days and months, no matter where they are in the world. (Of course, having a clear sky helps with the moon tracking!)

my notes

Earth orbits around the sun in approximately 365.2422 days. The sun's distance from a certain place on Earth determines the season: Fall, Winter, Spring, or Summer. If you wish your seasons to remain in sync with your calendar, you will probably want to have 365 or 366 days in a year. (This ensures January is always Winter in the Northern Hemisphere, not Spring or Fall or Summer.)

If we use the moon to create a calendar based on lunar months, the length of months varies between 29 and 30, so we will end up with a year that is more-or-less a multiple of 29.5: 29.5, 59, 88.5, 118, 147.5, 177, 206.5, 236, 265.5, 295, 324.5, 354, 383.5. In other words, we cannot have a lunar year that is exactly 365.2422 days. We *can* have a year that is 354 or 383.5 days, but if we do, the seasons will not be in sync with our seasons. (Sometimes January would be Spring, Fall, Summer, and yes, Winter in the Northern Hemisphere.)

Over thousands of years, humans have tried to make a calendar that accurately matches the sun and moon's relationships with Earth by creating a hack, more or less. We call that hack "leap day" or "leap month."

Earth's Orbit around the Sun	Moon's Orbit around Earth

CALENDARS & MATH

lunisolar

The Chinese, Hebrew, and Hindu lunar calendars all have leap months that occur on a scheduled basis. The Chinese system adds 7 leap months over the course of 19 years. The Hebrew one does as well. Yes, it's always a 19-year cycle, never 18 or 20 years. The Hindu calendar is a bit more complicated, as it varies based upon astronomers' observations of the moon.

- Aztec - combination of two different calendars (see below) into a calendar round
- Chinese
- Hebrew
- Hindu Panchanga calendar - combination of two different calendars (see below)

solar

- Aztec *Xiuhpohualli* (civil) calendar

  The Aztec solar calendar adds 13 days (one *trecena*) every 52 years. It's called the New Fire Ceremony and celebrates the binding of the calendars. Some other sources say the New Fire Ceremony was 12 days.

- Gregorian

  The Gregorian solar calendar adds one day every four years (but not always when it comes to century years like 2100). Yes, that is the calendar we use in most of the world today.

- Hindu solar (civil) calendar

lunar

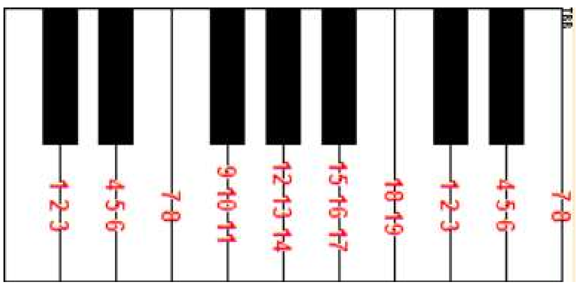
These calendars do not feature a "leap" adjustment to keep in sync with the seasons and the sun.

- Aztec *Tonalpohualli* (sacred) calendar
- Islamic
- Hindu sacred calendar

CALENDARS & MATH

lunisolar continued

Hebrew leaps



"[The leap month] is added in the 3rd, 6th, 8th, 11th, 14th, 17th and 19th years of the cycle."

Credit: jewfaq.org/calendar.htm

Hindu leaps

"Leap month occurs about once every three years...A month can either be added or omitted. An intercalary month, called Adhik Maas or Purushottam Maas, is added when a lunar month starts and ends before the Sun has moved to a new zodiac sign. In the rare case that the Sun traverses a whole zodiac sign during the course of a lunar month, the month is removed from the calendar. When this occurs, another month is repeated elsewhere in the year, so the year always has 12 or 13 months."

There are also leap days (added and omitted).

Credit: https://www.timeanddate.com/date/hindu-leap-year.html

Chinese leaps

The Chinese traditional calendar is based upon the moon and also tries to keep in sync with the seasons. It is deeply complex and based upon the observations of Chinese astronomers. Leap month occurs roughly every three years: seven times in 19 years. When exactly depends upon the astronomers.

Credit: https://www.timeanddate.com/calendar/about-chinese.html
https://www.timeanddate.com/date/chinese-leap-year.html

CALENDARS & MATH

Resources:

General

- https://www.dw.com/en/no-perfect-calendar-why-we-have-leap-years-and-why-the-earths-always-out-of-sync/a-19077064
- https://www.grc.nasa.gov/www/k-12/Numbers/Math/Mathematical_Thinking/calendar_calculations.htm
- https://www.timeanddate.com
- https://www.timeanddate.com/date/perfect-calendar.html
- http://www.webexhibits.org/calendars/

Aztec

- https://www.aztec.com/page.php?page=calendar
- https://www.mexicolore.co.uk/aztecs/calendar/the-aztec-or-central-mexican-calendar
- http://nahuatlstudies.blogspot.com/2017/04/the-aztecs-did-not-need-leap-year.html
- https://www.wired.com/2010/12/1217aztec-calendar-stone-rediscovered/

Chinese

- https://www.timeanddate.com/calendar/about-chinese.html
- https://www.timeanddate.com/date/chinese-leap-year.html
- http://www.webexhibits.org/calendars/calendar-chinese.html

Gregorian

- The Story of Clocks and Calendars by Betsy Maestro.
- https://www.timeanddate.com/calendar/gregorian-calendar.html

Hebrew

- http:// jewfaq.org/calendar.htm
- https://www.timeanddate.com/calendar/jewish-calendar.html
- http://www.webexhibits.org/calendars/calendar-jewish.html

Hindu

- https://medium.com/@wordcraftsolutions/what-is-so-right-about-the-hindu-calendar-708c85033b19
- https://www.timeanddate.com/calendar/hindu-calendar.html
- https://www.timeanddate.com/date/hindu-leap-year.html
- https://www.timecenter.com/articles/brief-history-of-the-hindu-calendar-by-timecenter/

Islamic

- https://www.timeanddate.com/calendar/islamic-calendar.html
- http://www.webexhibits.org/calendars/calendar-islamic.html

CALENDARS & MATH

Name: ________________

Date: ________________

Calendar	Total days per regular year	Total months per regular year	Days per month (range)	Leap what?
Aztec solar (civil)				
Chinese				
Gregorian /Christian				
Hebrew /Jewish				
Hindu lunar (sacred)				
Islamic				

MS. SWEET'S NOTES

Name: ________________

Date: ________________

Calendar	Total days per regular year	Total months per regular year	Days per month (range)	Leap what?
Aztec solar (civil)	365	18	20 days per month (and 5 extra days every year)	Every 52 years, the Aztec calendar adds 13 days.
Chinese	354ish (353, 354, or 355)	12	29-30	Leap month
Gregorian/ Christian	365	12	28-31	One leap day added every four years
Hebrew/ Jewish	354ish (353, 354, or 355)	12	29-30	Leap month
Hindu lunar (sacred)	354 or 355	12	29-30	Leap month
Islamic	354 or 355	12	29-30	N/A

CALENDARS & MATH

Name: ________________

Date: ________________

The Problem:

Which calendar is closest to the actual movement of Earth around the sun at the end of 19 years?

____

Required: Gregorian calendar & Earth's revolution around the sun

How many other calendars my teacher wants me to compare: ____

Circle the calendar(s) you are choosing:

Aztec Chinese Hebrew Hindu Islamic

CALENDARS & MATH

Name: ________________

Date: ________________

The Problem:

Which calendar is closest to the actual movement of Earth around the sun at the end of 19 years?

____

How many calendars my teacher wants me to compare: ____

Am I allowed to use a tablet/computer to make this chart? Circle: yes / no

Year #						Gregorian	Earth's revolution
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
Total days in 19 years							

SOLUTION

There is more than one correct solution since the Chinese, Hebrew, and Hindu calendars vary from year to year.

We used a spreadsheet program to create this table:

Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
1	365	354	354	354	354	365	365.2422
2	365	354	354	354	354	365	365.2422
3	365	384	384	383.5	354	365	365.2422
4	365	354	354	354	354	366	365.2422
10	365	354	354	354	354	365	365.2422
11	365	384	384	383.5	354	365	365.2422
12	365	354	354	354	354	366	365.2422
13	365	354	354	354	354	365	365.2422
14	365	384	384	383.5	354	365	365.2422
15	365	354	354	354	354	365	365.2422
16	365	354	354	383.5	354	366	365.2422
17	365	384	384	354	354	365	365.2422
18	365	354	354	354	354	365	365.2422
19	365	384	384	383.5	354	365	365.2422
	4.75						
Total days in 19 years	6939.75	6936	6936	6932.5	6726	6939	6939.6018

SOLUTION

Aztec calendar: Rather than adding 13 days every 52 years, we did 13/52*19 (4.75) and added that number after the 19^{th} year.

Chinese calendar: Because Chinese years vary from 353 to 355 days per year, we used 354 for all regular years. We did seven leap years in the 19 years, usually every third year. We started with two non-leap years followed by a leap year. If you start with a leap year or another point in the three-year cycle, your numbers will differ slightly. Because Chinese leap years vary from 383 to 385, we used 384 for all leap years.

Hebrew/Jewish calendar: Because Hebrew years vary from 353 to 355, we used 354 for all regular years. Because Hebrew leap years vary from 383 to 385, we used 384 for all leap years. We followed the leap year cycle as illustrated by the piano keyboard graphic.

Hindu lunar calendar: Because years vary, we used an average. A regular Hindu year is 354 days (29.5 days per month), and a Hindu leap year has one month, 29.5 days, more; this adds up to 383.5 days total. Leap years add a month; leap days can add or omit days. There is no established pattern, but it is every 2-3 years.

Islamic calendar: Because Islamic years vary from 353 to 355, we used 354 for all regular years. There is no leap year.

Gregorian/Christian calendar: Year four is the first leap year in our calculations. Every leap year is 366 days. If you start with a leap year or another point in the four-year cycle, your total will differ slightly since 19 is not divisible by four.

Earth's revolution: We used 365.2422 days per year for nineteen years. Instead of having a software program add them like we did, you could use multiplication: 365.2422 * 19.

SOLUTION

According to *this* work, the most accurate calendar after 19 years is...the Aztec calendar.

You can compare how far off these numbers are from the Earth's revolution answer using subtraction. In our software program, you can create a formula to do this for you. We recommend teaching students how to type the formula. Our formula happened to be "=$H$21 (Earth's total days in 19 years)-B21/C21/etc. (the calendar's total days in 19 years)".

SUM | =$H$21-C21

	A	B	C	D	E	F	G	H
1	Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
2	1	365	354	354	354	354	365	365.2422
3	2	365	354	354	354	354	365	365.2422
4	3	365	384	384	383.5	354	365	365.2422
5	4	365	354	354	354	354	366	365.2422
21	Total days in 19 years	6939.75	6936	6936	6932.5	6726	6939	6939.6018
22	Difference from Earth's revolution	-0.1482	=$H$21-C21					

After we copied the formula across the row (by clicking and dragging one time), it looked like this:

	A	B	C	D	E	F	G	H
1	Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
2	1	365	354	354	354	354	365	365.2422
3	2	365	354	354	354	354	365	365.2422
4	3	365	384	384	383.5	354	365	365.2422
5	4	365	354	354	354	354	366	365.2422
21	Total days in 19 years	6939.75	6936	6936	6932.5	6726	6939	6939.6018
22	Difference from Earth's revolution	-0.1482	3.6018	3.6018	7.1018	213.6018	0.6018	

According to *this*, as we said earlier, the Aztec calendar is the closest to the Earth's actual revolutions.

SOLUTION

However...

This small scale leads to varied results.

If your students start on different parts of the leap year cycles, they will get slightly different answers. This could be fertile ground for some great debates and a great reason to look at one another's work closely.

For example, if we start the Gregorian calendar with the leap year first:

	A	B	C	D	E	F	G	H
1	**Year #**	**Aztec**	**Chinese**	**Hebrew**	**Hindu**	**Islamic**	**Gregorian**	**Earth's revolution**
2	**1**	365	354	354	354	354	366	365.2422
3	**2**	365	354	354	354	354	365	365.2422
4	**3**	365	384	384	383.5	354	365	365.2422
5	**4**	365	354	354	354	354	365	365.2422
21	**Total days in 19 years**	6939.75	6936	6936	6932.5	6726	6940	6939.6018
22	**Difference from Earth's revolution**	-0.1482	3.6018	3.6018	7.1018	213.6018	-0.3982	

Some students will now want to expand into a larger number of years, which creates the extension on the next page.

EXTENSIONS

The Problem:

Which is most accurate: the Aztec calendar or the Gregorian calendar?

No peeking at the internet! You will feel so proud if you figure out the answer without doing an internet search. You can always search after you have found your own answer.

Extension A: Continue your spreadsheet to the full 52-year cycle of the Aztecs.

Extension B: Keep going to year 400 (because of the full Gregorian cycle of centuries being divisible by 400).

Extension C: Keep going to the first year that both 19 and 52 go into evenly, so all calendars cycles are complete (except that pesky Gregorian with its 400-year cycle).

Two Tips about Spreadsheets:

Using the spreadsheet formulas and click-and-drag to copy formulas makes these extensions possible to do in less than an hour (sometimes only a few minutes).

Another trick in our software program allows us to number rows or columns without typing all the numbers. For example, to number the years 20-52, highlight several numbers like 16-20 (A17-A21), then click and hold and drag down. The counting sequence should now appear where you dragged.

SOLUTION

Extension A:

We copied and pasted 19-year cycles of the Chinese, Hebrew and Hindu calendars; copied the Aztec and Islamic straight down, and then added 13 days to the last Aztec year. The Gregorian we copied and pasted the first 16 years and then deleted the extra to get back to year 52, which is divisible by 4 and therefore a leap year.

	A	B	C	D	E	F	G	H
1	Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
49	48	365	354	354	354	354	366	
50	49	365	384	384	383.5	354	365	
51	50	365	354	354	354	354	365	
52	51	365	354	354	354	354	365	
53	52	378	384	384	383.5	354	366	
54			354	354	354			
55			354	354	383.5			
56			384	384	354			
57			354	354	354			
58			384	384	383.5			

If we ignore the extra rows (54-58 above) for Extension A and add a row above them, we can use the SUM formula to calculate the total days in 52 years. For example:

	A	B	C	D	E	F	G	H
1	Year #	Aztec	Chinese	Hebrew	Hindu	Islamic	Gregorian	Earth's revolution
54	Total days in 52 years	18,993	18,978	18,978	18,969	18,408	18,993	18,993
55			354	354	354			
56			354	354	383.5			
57			384	384	354			
58			354	354	354			
59			384	384	383.5			

SOLUTION

Extension B:

We created a new sheet in our spreadsheet document, pasted Extension A into it, and then deleted the SUM row 54. We dragged and clicked column A down to row 401 to reach year 400.

Aztec: Then we copied the 52-year Aztec cycle and pasted it repeatedly.

Chinese, Hebrew, and Hindu: When we did Extension A, we already had copied the 19-year cycle four times (58 years). We copied those 58 years and pasted them until reaching 400 (almost).

Islamic: Straight copying of 354 using click and drag method.

Gregorian: We copied the first 50 years we had already done and pasted it down repeatedly. Then we had to go back to correct the century years as per Pope Gregory; otherwise we'd be like the Caesars. 100 divisible by 400? No, not a leap year. 200? No. 300? No. 400? Yes, a leap year.

At this point, we had no year 400 for the 19-year cycle lunar calendars because 19 goes evenly into 399. It was heartbreaking. Meanwhile, the Aztec calendar was still 17 years away from the end of a 52-year cycle.

	A	B	C	D	E	F	G	H
1	**Year #**	**Aztec**	**Chinese**	**Hebrew**	**Hindu**	**Islamic**	**Gregorian**	**Earth's revolution**
399	**398**	365	354	354	354	354	365	
400	**399**	365	384	384	383.5	354	365	
401	**400**	365					366	
402		365						
403		365						
404		365						
405		365						
406		365						
407		365						
408		365						
409		365						
410		365						
411		365						
412		365						
413		365						
414		365						
415		365						
416		365						
417		377						

SOLUTION

Extension B continued:

Then we again ignored the extra years (rows 402 and up), added a new row 402 (bumping the other rows down) to total the 400 years, using the SUM formula.

Rather than filling the "Earth's revolution" column, we made a formula in cell H402: =365.2422*400.

	A	B	C	D	E	F	G	H
1	**Year #**	**Aztec**	**Chinese**	**Hebrew**	**Hindu**	**Islamic**	**Gregorian**	**Earth's revolution**
402	**Total day in 400 years**	146,091	146,010	146,010	145,937	141,600	146,097	146,097

The Gregorian calendar is starting to look more accurate than the Aztec calendar, but that is because the Aztec calendar has not done its 12 extra days in the 52nd year of its cycle. We can't rule it out yet!

SOLUTION

Extension C

To find a year when the 19-year cycles will be complete and the 52-year cycle also will be complete, we do the LCM of 19 and 52. (If you are interested in learning more about applications of LCM, see the Sweet Math bundle "Aztec Calendar Math: Gears, Permutations, and LCM.")

The LCM of 52 and 19 is 988. Our software program can do this with a formula: =LCM(52,19). One can also do it using the prime factorization method--or simply multiplying 52 and 19 since 19 is prime (and 52 does not have 19 as a factor).

To do extension C efficiently, we copied Extension B into a new sheet, deleted row 402, and continued with similar methods of copying and pasting as before, always trying to copy as large a cycle as possible to avoid pasting a bazillion times.

At this point, it is difficult not to make an error somewhere. Our work has an error, and it's difficult to find.

According to this work, the Gregorian calendar is closest to Earth's revolution. However, there is an error in our work here. The Aztec calendar should be much closer than these calculations show it.

	A	B	C	D	E	F	G	H
1	**Year #**	**Aztec**	**Chinese**	**Hebrew**	**Hindu**	**Islamic**	**Gregorian**	**Earth's revolution**
990	**Total days in 988 years**	360,867	360,672	360,672	360,490	349,752	360,860	360,859
991	**Difference from Earth's revolution**	-7.71	187.29	187.29	369.29	11107.29	-0.71	

SOLUTION

Extension C continued:

According to timeanddate.com, the Mayan (and Aztec) calendar should be more accurate than the Gregorian calendar:

Calendar	Introduced	Average Year Length	Approximate Error
Persian calendar	2nd millennium BCE	365.2421986 days	Less than 1 sec/year (1 day in 110,000 years)
Revised Julian calendar	1923 CE	365.242222 days	2 sec/year (1 day in 31,250 years)
Mayan calendar	~2000 BCE	365.242036 days	13 sec/year (1 day in 6500 years)
Gregorian calendar	1582 CE	365.2425 days	27 sec/year (1 day in 3236 years)
Jewish calendar	9th century CE	365.246822 days	7 min/year (1 day in 216 years)
Julian calendar	45 BCE	365.25 days	11 min/year (1 day in 128 years)
Coptic calendar	25 BCE	365.25 days	11 min/year (1 day in 128 years)
365-day calendar (no leap years)*	-	365 days	6 hours/year (1 day in 4 years)

* There is no 365-day calendar system currently in use for civil purposes. Past examples include the ancient civil Egyptian calendar, the Maya Haab' calendar, and the Aztec Xiuhpohualli calendar.

Credit: https://www.timeanddate.com/date/perfect-calendar.html